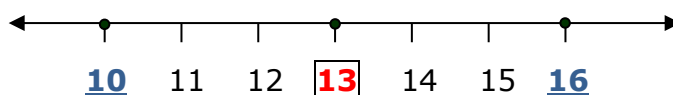

MIDPOINT ON THE LINE

AND IN THE PLANE

□ MIDPOINT ON THE LINE

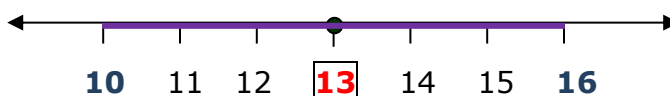
Here's a question for you: What number is *midway* between 10 and 16? You probably know that the number is 13. Why? Because 13 is 3 units away from 10, and 13 is also 3 units away from 16.



Now we need a simple way to find the number that is midway between any two numbers, even when the numbers are not nice, or worse yet, when there are variables involved. Notice: If we take the **average** (officially called the *arithmetic mean*) of 10 and 16 — by adding the two numbers and dividing by 2 — we get

$$\frac{10+16}{2} = \frac{26}{2} = 13, \text{ the midway number}$$

Let's rephrase what we've done with some new terminology. Consider the *line segment* connecting 10 and 16 on the number line:



We can now refer to the 13 as the ***midpoint*** of the line segment connecting 10 and 16.

What is the *midpoint* of the line segment connecting -2.8 and 14.6 ? Just calculate the average of -2.8 and 14.6 :

$$\frac{-2.8+14.6}{2} = \frac{11.8}{2} = 5.9$$

When you see the term ***midpoint***, think *average*!

Homework

1. Find the **midpoint** of the line segment connecting the two given numbers (or points) on a number line:

- | | | |
|--------------------|-------------------|---------------------|
| a. 10 and 20 | b. 13 and 22 | c. -8 and -26 |
| d. -3 and 7 | e. -7 and 6 | f. π and $-\pi$ |
| g. -21 and -99 | h. 0 and 43 | i. -50 and 0 |
| j. -44 and 19 | k. -41 and 88 | l. $3x$ and $-x$ |

□ MIDPOINT IN THE PLANE

Now for the more important question: Consider the two points $(2, 3)$ and $(6, 9)$ in the plane and the line segment that connects them. We need to figure out what point is the *midpoint* of the line segment connecting the two points.

Recall the advice given above:

When you see midpoint, think

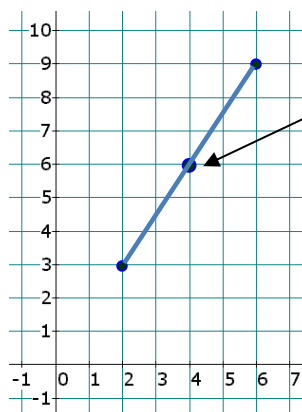
average. So the x -coordinate of the midpoint is the average of the x -coordinates of the two endpoints:

$$x = \frac{2+6}{2} = \frac{8}{2} = 4$$

And the y -coordinate of the midpoint is the average of the y -coordinates of the two endpoints:

$$y = \frac{3+9}{2} = \frac{12}{2} = 6$$

We conclude that the midpoint is **$(4, 6)$** . That's all there is to it. Now let's do a complete example without plotting any points or drawing any segments.



The **midpoint** is found by averaging the x -coordinates and averaging the y -coordinates.

EXAMPLE 1: Find the midpoint of the line segment connecting the points $(-42, -33)$ and $(90, -10)$.

Solution: No graphing needed — we have a formula. The x -coordinate of the midpoint is found by averaging the x -coordinates of the two given points:

$$x = \frac{-42 + 90}{2} = \frac{48}{2} = 24$$

The y -coordinate of the midpoint is found by averaging the y -coordinates of the two given points:

$$y = \frac{-33 + (-10)}{2} = \frac{-43}{2} = -\frac{43}{2}$$

The midpoint is therefore the point

$$\boxed{\left(24, -\frac{43}{2}\right)} \quad \text{or, } (24, -21.5)$$

Homework

2. Find the **midpoint** of the line segment connecting the given pair of points:
- | | |
|---|-------------------------------|
| a. $(-2, 5)$ and $(2, 7)$ | b. $(0, 1)$ and $(0, 6)$ |
| c. $(-5, 8)$ and $(-5, -8)$ | d. $(-2, 7)$ and $(5, -3)$ |
| e. $(-9, 2)$ and $(-13, -40)$ | f. $(0, 0)$ and $(-6, -9)$ |
| g. $(5, 4)$ and $(5, 4)$ | h. $(14, 0)$ and $(0, -9)$ |
| i. $(8, 8)$ and $(-19, -19)$ | j. $(\pi, 0)$ and $(-\pi, 0)$ |
| k. $(0, \sqrt{2})$ and $(0, -\sqrt{2})$ | l. (a, b) and (c, d) |
| m. (a, b) and $(a, -b)$ | n. $(3a, 3b)$ and $(-3a, b)$ |

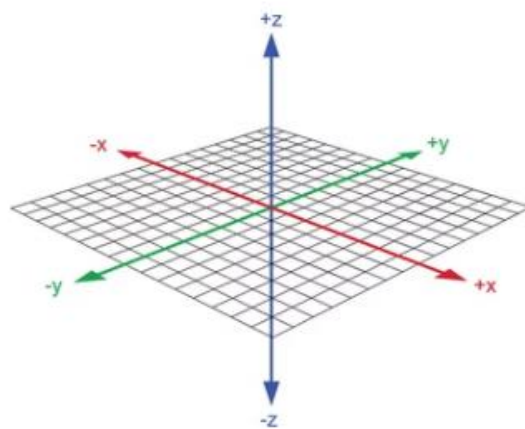
❑ TO ∞ AND BEYOND

A point on a 1-dimensional line can be described by a single number, for example, 7.

A point in a 2-dimensional plane can be described by an ordered pair, for example, $(2, -9)$.

A point in 3-dimensional space (which has an x -axis, a y -axis, and a z -axis), can be described by an ordered triple, for example $(1, -5, 12)$.

Find the **midpoint** of the line segment connecting the points $(2, -3, 7)$ and $(-5, 17, 20)$.



Solutions

1. a. 15 b. 17.5 c. -17 d. 2 e. -0.5 f. 0
g. -60 h. 21.5 i. -25 j. -12.5 k. 23.5 l. x
2. a. $(0, 6)$ b. $(0, 7/2)$ c. $(-5, 0)$
d. $(3/2, 2)$ e. $(-11, -19)$ f. $(-3, -9/2)$
g. $(5, 4)$ h. $(7, -9/2)$ i. $(-11/2, -11/2)$
j. $(0, 0)$ k. $(0, 0)$ l. $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$
m. $(a, 0)$ n. $(0, 2b)$

"What sculpture is to a block of marble,
education is to the human soul."

Joseph Addison

